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Reply to comment by J. Trampert

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In response to the comment by Trampert (1990) on the paper 'Comparison of iterative back-projection inversion and generalized inversion . . .' by Ho-Liu, Montagner & Kanamori (1989; hereafter referred as HMK), we would like to reiterate the two main objectives of HMK.

The first one was to compare the generalized least-squares inversion method without blocks by Tarantola & Valette (1982; hereafter referred to as TV) and a particular version of SIRT technique (Van der Sluis & Van der Vorst 1987) presented by Comer & Clayton (1985, unpublished manuscript; hereafter referred to as CC). We did not intend to provide a general relationship between TV and the general class of SIRT. We wanted to provide a way to choose the damping factor in CC using an approximate form of back projection algorithm (HMK, p. 22), and to assess the error and resolution. In the previous studies using CC, the damping factor was chosen more or less arbitrarily. The second purpose was to apply these two different tomographic techniques to the same data set to check the robustness of the inversion results.

H. Trampert notes that two points need clarification. We will consider successively these two points. The first point concerns the damping factor μ. Equation (1) from TV as reported by Trampert is the general algorithm solving a non-linear problem. We applied it to a linear problem (section 6 of HMK) without iteration (it is clearly pointed out in section 4 of HKM). We did not solve a non-linear problem. Comparing the two algorithms, we only showed that CC (actually slightly different from SIRT) cannot solve the problem by one iteration because CC's expression is approximate. The correction term $\delta\mathbf{p}_{\mathrm{CC}} = p_{k+1} - p_k$ of CC is always smaller than the exact term given by TV, $\delta\mathbf{p}_{\mathrm{TV}} = p_1 - p_0$. Consequently, it is necessary to iterate when CC is used, in order to converge. That is the reason why we have kept an index k for $\delta\mathbf{p}_{\mathrm{CC}}$. To avoid confusion, we could have used different indices for TV and CC. Trampert concludes that '*As a consequence, their interpretation of μ as the ratio between the data variance and the* a priori *model variance is incorrect.*' This statement is odd because our expression (17)

$$\mu = \frac{\sigma_{\mathrm{d}}^2}{\sigma_{\mathrm{p}}^2} \tag{R1}$$

is the same as the damping factor of Trampert & Lévêque (1990) in a particular case. They are right, however, in that the weighting of the data by $1/\sqrt{L}$ (where L is the length of each path) can be dangerous (problem of rescaling). This is the reason why we pointed out in HKM that it was necessary to determine the damping factor empirically for each case.

The second point concerns the calculation and resolution in SIRT. We have never said that it is impossible to compute an error on parameters from SIRT. We only said that it was not available to us; we are pleased to learn that Trampert & Lévêque (1990) are able to derive such expressions. However, the expression of resolution for TV was first given by Montagner & Jobert (1981) and the relation (5) about $\mathbf{C}_p$ is given explicitly in Montagner (1986). The real problem was a practical one. That is the reason why numerical tests were performed in order to assess the reliability of the inversion by SIRT (Humphreys & Clayton 1988; Ho-Liu *et al.* 1989). We used different methods to compute resolution and errors. But, to our knowledge, the exact way to do that is the one proposed by TV.

In conclusion, we think that contrary to the assertion of J. Trampert, there is no conceptual mistake in our way of relating SIRT and TV. Our paper is more concerned with data interpretation than theoretical considerations. We are happy to see that Trampert & Lévêque (1990) have been able to obtain the relations between TV and a large class of SIRT techniques.

REFERENCES

Ho-Liu, P., Montagner, J. P. & Kanamori, H., 1989. Comparison of iterative back-projection inversion and generalized inversion without blocks: case studies in attenuation tomograpy, *Geophys. J. R. astr. Soc.*, **97**, 19–29.

Humphreys, E. & Clayton, R. W., 1988. Adaptation of back projection tomography to seismic travel time problems. *J. geophys. Res.*, **93**, 1073–1085.

Montagner, J. P., 1986. Regional three-dimensional structures using long period surface waves, *Ann. Geophys.*, **4**, 283–294.

Montagner, J. P. & Jobert, N., 1981. Investigation of upper mantle structure under young regions of the South-East Pacific using long period Rayleigh waves, *Phys. Earth planet. Inter.*, **27**, 206–222.

Tarantola, A. & Valette, B., 1982. Generalized nonlinear inverse problems solved using the least squares criterion, *Rev. Geophys. Space Phys.*, **20**, 219–232.

Trampert, J., 1990. Comment on 'Comparison of iterative back-projection inversion and generalized inversion without blocks: case studies in attenuation tomography' by P. Ho-Liu, J.-P. Montagner and H. Kanamori, *Geophys. J. Int.*, **103,** 755–756.

Trampert, J. & Lévêque, J. J., 1990. SIRT: Physical interpretation based on the generalized least-square solution, *J. geophys. Res.*, in press.

Van der Sluis, A. & Van der Vorst, H. A., 1987. Numerical solution of large, sparse linear algebraic systems arising from tomographic problems, in *Seismic Tomography*, pp. 49–83, ed. Nolet, G. D., Reidel, Dordrecht.